Reactive Task Planner Synthesis of Multi-Contact Dynamic Locomotion in Constrained Environments

Ye Zhao*, Ufuk Topcu**, and Luis Sentis*
*Human Centered Robotics Laboratory, The University of Texas at Austin, USA
**Department of Aerospace Engineering and Engineering Mechanics,
The University of Texas at Austin, USA
Email: *yezhao@utexas.edu, utopcu@utexas.edu, lsentis@austin.utexas.edu*

***Abstract*—Contact-based decision and planning methods are increasingly being sought for task execution in humanoid robots. Formal methods for verification and synthesis have not been yet incorporated into the motion planning sequence to generate complex mobility behaviors in legged humanoid robots. This study takes a step toward formally synthesizing a high-level reactive task planner for unified legged and armed locomotion in constrained environments. We formulate a two-player temporal logic game between the contact planner and its possibly adversarial environment. The resulting discrete planner satisfies the given task specifications expressed as a fragment of temporal logic and is executed by a low-level phase-space planner. We devise a set of low-level locomotion modes based on centroidal momentum dynamics. Provable correctness of the low-level execution of the synthesized discrete planner is guaranteed through the so-called simulation relations. Simulations of dynamic locomotion in constrained environments support the effectiveness of the outlined hierarchical planner protocol.**

***Index Terms*—Multi-contact locomotion, Task and motion planning, Temporal logic, Reactive synthesis, Switched system.**

I. INTRODUCTION

Many planning methods for mobility in humanoid robots are designed and implemented in the continuous space or the hybrid space with a concentration on the continuous part. In contrast with existing techniques, we investigate formal methods for discrete task planner synthesis of humanoid robot behaviors [1, 2]. Although widely used in the mobile robotics motion planning community [3, 4, 5], formal verification and synthesis methods are still under-exploited within the locomotion community. A potential reason is that humanoid robots possess high-dimensional and under-actuated dynamics. To circumvent this difficulty, our work in [6, 7] proposed a phase-space planning framework that leverages low-dimensional models for the dynamic locomotion process while still characterizing its intrinsic dynamics. In particular, center-of-mass trajectories in the phase-space are sequentially composed based on determining locomotion keyframe states.

Recently, formal methods have started to be used for humanoid robots, such as robotic manipulations [8, 9, 10]. However, formal methods are yet to be explored for legged locomotion, or for more complex multi-contact locomotion tasks. The authors in [11] implement a partially observable Markov decision process (POMDP) to account for terrain uncertainties and make sequential decisions for locomotion controller switching. However, this work is limited to mildly rough terrains. In contrast, our environments are composed of highly rough and constrained environments with emergent events, which needs responsive locomotion behaviors.

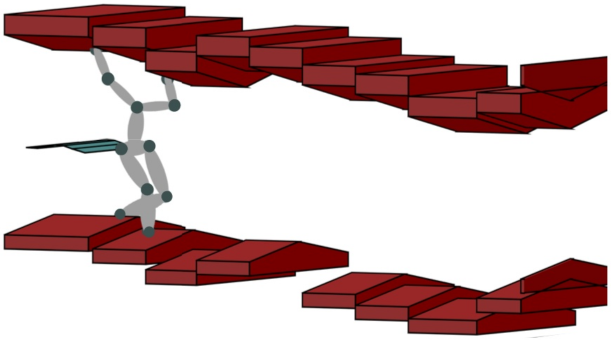

Fig. 1: Maneuvering in a constrained environment via multi-contact legged and armed locomotion.

II. CONTRIBUTION

Our objective is to synthesize a correct high-level reactive planner for the multi-contact locomotion problem. Fig. 1 shows an example scenario motivated by a naval research application: a multi-limb robot maneuvers and operates within a constrained environment such as a submarine vessel. The main contribution of this work is to devise a high-level reactive task planner for the multi-contact locomotion by solving a two-player game [12]. Environment actions are treated as adversaries in this game. We employ linear temporal logic (LTL) to specify multi-contact locomotion behaviors. In particular, we focus on the communication between the high-level task planner and low-level motion planner via switching signals and the correctness of the interplay. We rely on a discretization of the phase-space for keyframe state design. As a generalization of rough terrain locomotion [7, 13], we focus not only on the multi-contact behaviors but also on its response to unexpected environmental events such as stair crack and human appearance in the scene. To the best of our knowledge, this study is the first attempt to use formal methods for multi-contact locomotion behaviors and task planner synthesis with

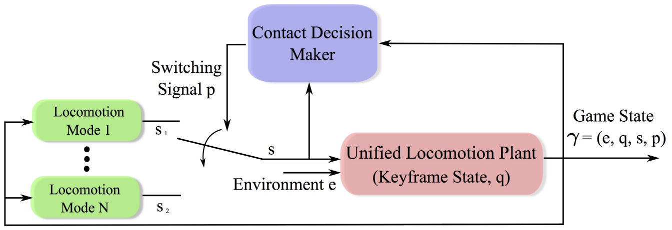

Fig. 2: Logic-based planner structure. Each mode, indexed by a switching signal p, corresponds to a locomotion model. Four modes are modeled for the maneuvering in constrained dynamic environments. The environment action is represented by e while the control action is represented by s describing the limb contact configuration. The discretized phase-space keyframe is $q = (p_{\text{contact}}, \dot{x}_{\text{apex}})$.

guarantee of correctness. In the light of increasing demands for sophisticated humanoid contact tasks, it is imperative to devise high-level abstracted planners for reactive contact decisions. We expect that this line of work acts as an entry point for the humanoid robotics community to employ formal methods to verify and synthesize task planners.

III. SWITCHED DYNAMICS AND TASK SPECIFICATIONS

Dynamics of the multi-contact locomotion can be defined as a switched system with a switching signal p indexing specific locomotion modes. A logic-based switched system is shown in Fig. 1. Given a sequence of switching signals, the low-level planner evolves continuously in one mode and switches to the next one based on the computed contact transitions. To accomplish multi-contact locomotion behaviors, we compose a sequence of locomotion modes with planned keyframes. This is achievable by synthesizing a high-level planner protocol which makes proper decisions on limb contacts and low-level planner switchings. Given these preliminaries, we formulate a discrete planner synthesis problem.

Discrete Contact Planner Switching Synthesis: Given a transition system $\mathcal{TS}$ and a LTL specification φ following an assume-guarantee form [12], we synthesize a contact planner switching strategy that generates only correct executions $\gamma = (q, p, e, s)$.

To make the computation tractable, we employ a class of LTL formulae with favorable polynomial complexity, named as the Generalized Reactivity (1) formulae [12]. Based on this formulae, we design LTL task specifications involving environment actions, system actions and keyframe state design.

IV. HIGH-LEVEL REACTIVE PLANNER SYNTHESIS

Given a set of task specifications, we synthesize a reactive planner by formulating the high-level multi-contact locomotion planning problem as a game between the robot and its environment. This locomotion planner game is a tuple $\mathcal{G}$, composed of input and output variables, initial states, transition relations and a LTL winning condition [12]. Given this game, a winning strategy of the switched system for the pair $(\mathcal{TS}, \varphi)$ is defined as a partial function $(\gamma_0\gamma_1 \cdots \gamma_{i-1}, (q_i, e_i)) \mapsto (s_i, p_i)$, where a contact configuration s_i and a switching mode p_i are chosen according to the state sequence history, the current keyframe state q_i and environment action e_i. All the specifications are satisfied whatever admissible yet uncontrollable environment actions are. *A winning multi-contact locomotion strategy exists for the game $\mathcal{G}$ if and only if $(\mathcal{TS}, \varphi)$ is realizable.* Fig. 3 shows an automaton fragment of the locomotion contact planner.

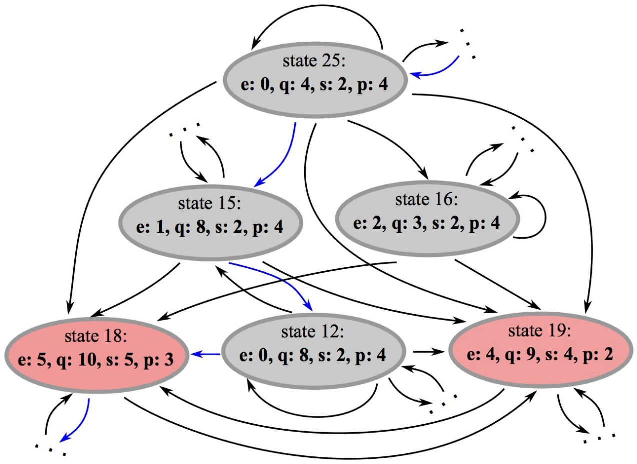

Fig. 3: A fragment of the synthesized automaton for the multi-contact planner. Nondeterministic transitions are encoded in this automaton. The blue transitions represent a specific execution. For notation convenience, the environment action e, keyframe state q, system action s and switching signal p are all indexed by numbers. For instance, when the state is at 25, we have $e = 0$ and $q = 4$. Then the winning strategy assigns $s = 2$ and $p = 4$.

To guarantee the correctness of the hierarchical planning protocol, we define a mapping between the low-level trajectory and high-level execution, and prove: *Given an over-approximation model, a winning multi-contact locomotion strategy synthesized from the two-player game is guaranteed to be correctly implemented by the underlying low-level phase-space planners.*

V. LOW-LEVEL MOTION PLANNER MODES

Given the high-level discrete planner, we design a low-level phase-space planner that consists of a set of locomotion modes, and compose them sequentially to guarantee the correctness of overall implementation. To begin with, we introduce centroidal momentum dynamics in a general sense. Dynamics of mechanical systems can be represented by their rate of change of linear and angular centroidal momenta, which are affected by external wrenches (force/torque) exerted on the system. Given this general model and mild assumptions, we propose four specific locomotion modes according to specific multi-contact configurations: mode (a) prismatic inverted pendulum model, mode (b) pendulum model, mode (c) stop-launch model, and mode (d) centroidal-momentum-based multi-contact model. Phase-space trajectories of these modes are shown in the top subfigures of Fig. 5. Given a mode switching signal p commanded from the high-level planner, the low-level phase-space planner generates the continuous CoM trajectory and computes the mode switching event, i.e., walking step transition.

VI. RESULTS

The locomotion tasks are achieved by combining the synthesized planner and the low-level planners via switched

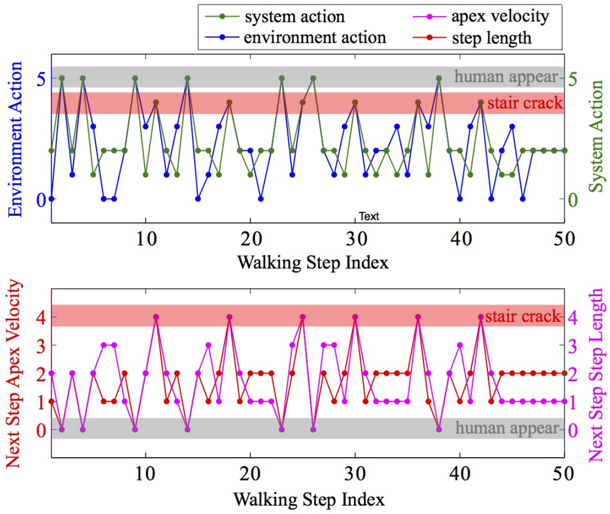

Fig. 4: Environment actions, system actions and keyframe states of 50 walking steps according to the synthesized automaton. Actions and states are indexed by numbers. Unexpected events, i.e, human appear and stair crack, are highlighted in the shaded regions. In the second subfigure, the numbers 0-4 on the vertical axis correspond to $\{0, 0.4, 0.6, 0.8, 1.7\}$ m/s for next step apex velocity and $\{0.15, 0.5, 0.6, 0.7, 0.6\}$ m for next step step length.

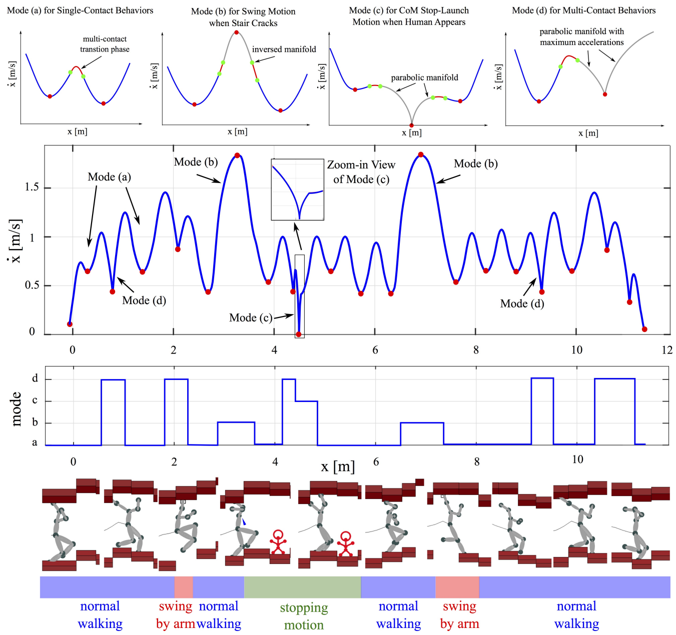

Fig. 5: CoM sagittal phase-space trajectories, mode switchings and motion snapshots for a 20-step locomotion process. The top four figures illustrate phase-space manifolds of different locomotion modes. The mode switching is governed by the high-level contact planner. Emergent behaviors are incorporated, i.e., two stair crack and one human appearance as shown in the locomotion snapshots at the bottom.

modes. Keyframe state $\mathcal{Q}$ is decomposed into two states: apex velocity and step length. For either state, Small, Normal and Large labels are assigned to $\{1, 2, 3\}$ in order while Stop and Swing labels are assigned to 0 and 4, respectively. The Temporal Logic Planning (TuLiP) toolbox, a python-based embedded control software [12], is used to synthesize the high-level contact planner. Our resulting high-level planner is represented by a finite state automaton with 27 states and 148 transitions in total. Fig. 4 illustrates discrete environment and system actions, and their corresponding keyframe states. As it shows, the system actions and modes switch correctly according to non-deterministic environment actions and keyframe states. For the low-level planners, four modes as shown in Fig. 5 are alternated according to the high-level switching protocol. Fig. 5 illustrates a synthesized CoM sagittal phase-space trajectory and snapshots of a 20-step dynamic walking. An accompanying video is available at https://youtu.be/urp7xu8vx3s.

VII. ONGOING WORKS

Our current works are leveraging the proposed planner synthesis to more generalized locomotion behaviors, such as maneuvering in cluttered environments. Other practical issues, such as contact reachability and robot actuation limits, are also being studied. In the future, we plan to incorporate probabilistic models, such as Markov decision process or POMDP. Reasoning uncertainties and quantifying robust performance at the abstract level are also of our interest.

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